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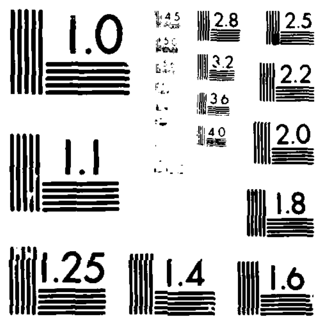

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(6) DELTA-EXACT LOWER CONFIDENCE BOUNDS
FOR SERIES AND PSEUDO-SERIES SYSTEM RELIABILITY.

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Methods of computing lower confidence bounds are examined for applicability to actual data and systems. Improvements to a modification of the likelihood Ratio method are shown for the series case and a variety of methods are studied for pseudo-series, which may model some missiles.

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I. BACKGROUND

Many expensive systems, such as missiles, are produced to complete one task only and they must be certified to the purchaser to be highly reliable even though they may not be tested directly. In such a case, the producer will test individual components or subsystems of the system and obtain a point estimate from the resulting binomial data: x_i successes out of n_i trials for $1 \le i \le k$, where k is the number of components. Further, most contracts require an interval estimate in the form of a lower confidence bound on system reliability. The interval estimator is much harder to produce: Even though the literature contains a number of methods for computing interval estimates for the reliability of series systems, these methods often do not produce realistic bounds for highly reliable systems where the data consist of disparate component sample sizes and a paucity of failures.

Although there are a number of properties of lower confidence bounds that have been described historically in the mathematical literature as desireable, unbiased, uniformly most accurate, optimum, exact, consistent [7], the methods presently available are approximations so these properties are difficult to verify except for special, unrealistic cases. Properties which are desireable and that can be tested by computer simulation are: a method produces a δ-exact$(1-\alpha)\%$ lower confidence bound if in the long run $(1-\alpha)\%$ of the bounds are less than the true reliability plus δ [7] ($\delta=0$ corresponds

to an exact bound). The method should produce bounds that are as close to the true reliability as possible: the variance of the confidence bounds about the true reliability should be minimal. The method should be flexible so that it can be applied to the variety of cases that arise in practice: The constraints of time and expense often result in test data from a variety of sample sizes, n_i, $1 \le i \le k$, with the sample sizes of all or many components small either absolutely or with respect to the other sample sizes. The method should be applicable when some or all of the components experience no failures in testing.

In [7], work was focused on series systems as this is the simplest case with the most existing lower confidence bound methods. There are presently many methods used to compute lower confidence bounds for series systems: modified maximum likelihood (MMLI) introduced by Easterling [2]; approximately optimal (AO) presented by Mann and Grubb [9]; modified log gamma (MLG) introduced by Borsting and Woods [1] and recently modified by Tomsky and by the authors to remove an inconsistency (see [7] for further discussion of these methods); the likelihood ratio methods discussed below. However, each method has difficulties in some practical cases, so the concept of δ-exact bounds was introduced [7] to allow a larger class of methods to be considered. It was shown that some new methods which are δ=.01 90% lower confidence bounds seem to be preferrable in many practical

cases.

In 1965, Madansky [8] presented a method for series and parallel systems that was based on the result, due to Wilks, that $-2\ln(P_0)$ is distributed asymptotically as χ^2 with one degree of freedom. In 1968, the method was extended to all coherent systems by Myhre and Saunders [11]. As originally introduced, this method had difficulties when the system was so reliable or the samples taken so small that some components were tested without a failure resulting. As this is often the case for expensive highly reliable systems, such as missiles, an initial attempt to correct this difficulty was made by adjusting the number of failures to $4/kn_i$ for the i-th component when there were no actual failures out of n_i trials for the i-th component. The adjusted binomial data is then used by the likelihood ratio method resulting in a method referred to as LRL.

In the past, the likelihood ratio method has used the maximum likelihood estimate of the reliability of the system components. However, in terms of total squared error loss, Stein found [6] that one could improve on the MLE when estimating three or more parameters from independent normal observations and Efron and Morris have shown how to extend this estimator to binomial data [6]. By using Stein's estimates of the component reliability in the likelihood ratio method, a potentially acceptable method was produced though it tended to be too optimistic in certain situations. Since Stein's

estimator shrinks the estimated value parameters from the MLE toward the mean of the prior distribution, it gives poor estimates of the parameter of individual components that have unusually large or small reliability. What is needed for such cases is a compromise between the Stein's estimator and the MLE. That compromise consists of rules which limit the amount of deviation the Stein's estimator can take from the MLE; these rules are known as limited translation rules [4]. These rules reduce the total risk that results from using the MLE, but have a greater total risk than Stein's estimator. Therefore, the savings of risk of a limited translation rule lies between 0, for the MLE, and 1, for the Stein's estimator. In our previous report, LRS13 was introduced, it being a limited translation version of Stein's estimator with 80% savings followed by the likelihood ratio method.

Since the component test data is binomially distributed and not normally distributed, an empirical Bayesian approach was applied directly to the binomial data, assuming a Beta prior distribution. (See [7] for more details.) The adjusted empirical Bayesian binomial test data is used in the likelihood ratio method resulting in the method LRB9.

In our previous report, it was shown that the likelihood ratio methods LRS13 and LRB9 have great promise especially in the unequal sample size case but also for the small equal sample size case, exactly the most troublesome case for other methods. LRS13 was not always a δ-exact bound, but since the 80% level of savings used in LRS13 was chosen arbitrarily,

further work was warrented to determine if there was a level of savings that would produce a better method for either the equal or unequal sample case.

While the previous work was confined to the series systems, another type of system, referred to as a pseudo-series, is very important as it can be used to estimate the reliability of a missile shot from a submarine and having re-entry bodies successfully ejected from it. Making a few assumptions included below, this might be modeled as: $R=R_E R_{FS} R_{SS} R_{TS} R_{I1} R_I^{n-1/2} R_V$, where R is system reliability; R_E is the probability that a missile is successfully ejected from the submarine; R_{FS}, R_{SS}, R_{TS} are the probabilities that a missile which enters the first, second, or third boost phase will not fail during it (naturally, R_{FS} is conditioned on the completion of the first boost phase, ...); R_{I1} is the probability that a missile which enters the horizontal deployment phase will not fail before the predetermined time of the first re-entry body release; R_{Ij} is the probability that the missile continues to function between the time of the (j-1)-th and j-th planned release of a re-entry body ($2 \le j \le N$, where N is the number of re-entry bodies on the missile). It is assumed that the R_{Ij} are equal for all j above, so R_{Ij} is replaced by R_I. R_V is the probability of success of the portion of the missile which is concerned only with the ejection of a single re-entry body, and R_V is assumed to be constant for all re-entry bodies. Thus, the reliability function is: $H(p)=\prod_{i=1}^{k} p_i^{\alpha_i}$, where $\alpha_i=1$ except for one component.

If the set of missiles has been purchased from different vendors or gone through re-designing, then the reliability function will be $H(p)=\sum_{j=1}^{m} \omega_j \prod_{i=1}^{k} p_i^{\alpha_{ij}}$. Here, ω_j represents the proportion of missiles from vendor j, for instance. So far, only MLG and the likelihood ratio methods have been shown to work for pseudo-series.

In the past few months, we have concentrated on two problems: 1) What is the best level of savings to use in a likelihood ratio method such as LRS13? Is 80% optimum in some cases, or can the method be improved by using 50%, 65%, or 95% savings? 2) Among all methods of estimating lower confidence bounds for the reliability of pseudo-series, which are best? Do the likelihood ratio methods that worked well for series work as well for pseudo-series?

II. METHODS OF SIMULATION AND COMPARISON

Our simulations were done as explained in our preliminary report. As was explained there, we designate a number, k, of components which are in a structure (in our case, a series or pseudo-series structure). For each component i, $1 \leq i \leq k$, we assign a reliability p_i and specify the number n_i of parts to be tested. We then simulate success-failure data for each component and compute lower confidence bounds on the system reliability using various confidence bound techniques. This process is repeated 100 times and the results are recorded

for each lower confidence bound method which was considered.

For the cases with series systems, we compared six different limited translation versions of Stein's estimator which adjusted data and fed it into the likelihood ratio method. In our previous studies of series systems, this seemed a promising method of estimating lower confidence bounds. For the pseudo-series system, we considered five methods which had seemed promising (one of which was a Stein's method).

After the lower bounds have been computed, the results of each lower confidence bound method are compared with the true system reliability (found by plugging the p_i's into the reliability function) to see how often each method produced a confidence bound which was no greater than the true system reliability. In computing 90% lower confidence bounds, we desire no more than 10% of the computed bounds to be greater than the true system reliability. We also consider δ-exact bounds by comparing the results against the true reliability plus δ, where δ=.01 and .02.

We consider how close the results of a method are by computing the standard deviation of the 100 results about the true reliability. We also compute the value measured about the true reliability plus δ, where again δ=.01 and .02. In our original report, we considered only the value computed about the true reliability, but we feel it is necessary to use one of the other values when a method seems to work as a δ=.01 or .02 bound.

In our preliminary report, we compared lower confidence bound methods pairwise by comparing the type of bound each method produced (an exact bound, δ=.01 or .02 bound, or none of these),

and the standard deviation of each method's results about the true system reliability. We also counted the number of sets of 100 simulations where each method produced each type of bound. Both types of results are necessary in evaluating and comparing methods, but we found that one type of result could contradict the indication of the other type. Below we see an excerpt from the preliminary report which is part of Table VIII of that report and which shows the results of series simulations for systems with k=20 components. We see that with pairwise comparisons, the method LRS13 looked like the best method beating out every other method (the rightmost column tallies how many methods it beat out versus the number it lost to: 5-0). Looking at the other part of the table however, we see that LRS13 was not a bound 3 out of 7 times.

	best overall	δ-exact bound δ=0(exact)	δ=.01	δ=.02	not a bound
AO		6	1		
MMLI		7			
MLGB		5	1		1
LRL		4	1	1	1
LRS13	4	3	1		3
LRB9	3	7			

Pairwise Comparison of Methods for the Sets of Simulations	AO	MMLI	MLGB	LRL	LRS13	LRB9	Comparison of Individual Methods with all Others
AO	-	7-0	2-5	3-4	3-4	0-7	1-4
MMLI	0-7	-	1-6	2-5	3-4	0-7	0-5
MLGB	5-2	6-1	-	7-0	1-6	2-5	3-2
LRL	4-3	5-2	0-7	-	2-5	0-7	2-3
LRS13	4-3	4-3	6-1	5-2	-	4-3	5-0
LRB9	7-0	7-0	5-2	7-0	3-4	-	4-1

In the past few months, we have spent time looking for a method of evaluating and comparing methods which would not produce contradictory results as the previous method could. First we considered specifying each method as some specific type of bound (exact, $\delta=.01$,$\delta=.02$, or none of these) by comparing how often (the number of sets of simulations) it produced each type of bound. If a method produced an exact bound at least 90% of the time, we called the method an exact method. If the method was not an exact method, but produced either an exact or a $\delta=.01$ bound at least 90% of the time, we called it a $\delta=.01$ method. We similarly defined a $\delta=.02$ method. If a method could not be called any of these, it was always considered a $\delta=.02$ method for comparison purposes. We compared results pairwise by considering whether each of the 2 methods produced a bound at least as good as the one it was specified as. If so, we compared the standard deviations corresponding to the specified bounds and the method with the lower value would win. If neither produced the specified bound, the smallest again won. And if only one produced its specified bound, that one automatically won out. While this method seemed to have advantages, some inconsistancies still showed up.

The way we finally decided to compare methods was to specify each method as a certain type of bound as discussed above, and then compute the average standard deviation, over the set of simulations, for each method, where the standard deviations compared were those computed for the type of bound

which the method was specified. These averages could be compared from method to method with smaller values indicating a better method. Thus, a method which produced a δ=.01 bound could be better than another method which produced an exact bound simply because on the average it produced results which were closer to the true system reliabilities. If a method did not produce any type of bound, it automatically lost out to any method which did. It could however, be compared to other methods which did not produce any type of bound. In the next sections, we report the results we accumulated.

III. SERIES RESULTS

In our previous work [7], we found that the results differed between the case of series systems with equal component test sample size and the case of unequal sample sizes. The LRS13 method showed great promise for the unequal sample size case with noticeably smaller sample standard deviations about the true reliability than the other methods available. However, LRS13 was so optimistic that it did not often produce exact 90% bounds and instead was most consistently a .02-exact 90% bound. In the equal sample size case, this was even more pronounced with LRS13 often failing to be any type of 90% bound. As mentioned above, LRS13 used a limited translation version of Stein's estimator to adjust the test data before feeding it into the likelihood ratio method and this version has an 80% level of savings. In [4], tables are

given for the constant, D, necessary to achieve six levels of savings: .5, .6, .75, .8, .9, .95 and the different levels of savings offered hope for improvements in LRS13.

Using these levels of savings, six methods, LRSS1, LRSS2, LRSS3, LRSS4, LRSS5, and LRSS6, were developed and tested for a variety of cases. The set of simulations covers the cases with k=6, 10 or 20 components; system reliability of .85, .88, .90, .93, and (for k=6) .95; and equal and unequal sample sizes. For comparison purposes, we looked first at the percentage of time the method was cumlatively a δ=.00, .01 and .02 bound, and then looked at the corresponding average sample δ-standard deviation for each method. These results are grouped by system size and type of sample size in Tables 1-6. Tables 1-3 contain the results for unequal sample sizes (5,10,20,30), (15,30,60), and (20,30,40,50). Tables 4-6 contain the results for equal sample sizes (15,20) and (30,40).

	LRSS1	LRSS2	LRSS3	LRSS4	LRSS5	LRSS6
Exact	6 (.1425)	6 (.1420)	8 (.1416)	8 (.1415)	8 (.1417)	8 (.1407)
.01	9 (.1505)	9 (.1501)	9 (.1498)	9 (.1499)	10 (.1502)	10 (.1492)
.02	11 (.1589)	11 (.1585)	11 (.1584)	11 (.1584)	12 (.1589)	12 (.1579)
Not a Bound	1	1	1	1	0	0
Rank	6	4	2	2	5	1

TABLE 1: k=6

Unequal Sample Sizes

	LRSS1	LRSS2	LRSS3	LRSS4	LRSS5	LRSS6
Exact	5 (.1461)	6 (.1467)	7 (.1468)	9 (.1458)	10 (.1459)	10 (.1437)
.01	8 (.1542)	8 (.1537)	12 (.1539)	12 (.1542)	11 (.1545)	11 (.1524)
.02	10* (.1625)	11 (.1625)	12 (.1625)	12 (.1629)	12 (.1632)	12 (.1612)
Not a Bound	2	1	0	0	0	0
Rank	6	5	2	3	4	1

TABLE 2: k=10
Unequal Sample Sizes

	LRSS1	LRSS2	LRSS3	LRSS	LRSS5	LRSS6
Exact	5 (.1434)	5 (.1445)	7 (.1488)	8 (.1518)	9 (.1614)	9 (.1640)
.01	7 (.1515)	7 (.1527)	9 (.1574)	9 (.1605)	9 (.1705)	9 (.1732)
.02	8 (.1598)	9 (.1602)	9 (.1662)	9 (.1695)	9 (.1798)	9 (.1826)
Not a Bound	1	0	0	0	0	0
Rank	6	5	1	2	3	4

TABLE 3: k=20
Unequal Sample Sizes

○ - Designated Type of Bound (90%)
* - Not Really a .02 Bound
( ) - Average Standard Deviation

In Table 1, the results for the unequal sample size case and k=6, we can see that LRSS6 is the preferable method

since it is a .02-exact bound 12 out of 12 times, the other methods are also .02-exact bounds, and the average sample .02-standard deviation of LRSS6 is the smallest; however, there is little difference among these averages. In Table 2, the unequal sample case for k=10, LRSS6 is again better than the other methods as it is a .01-exact 90% bound 11 out of 12 times and its average sample .01-standard deviation is smaller than the allowable average sample standard deviation for any of the other methods. In this case there is a significant difference between the δ-standard deviations of the best and worse methods. In Table 3, the unequal sample case for k=20, LRSS3 and LRSS4 are the top ranked methods with LRSS5 and LRSS6 appearing overly conservative but definitely exact 90% bounds. Thus, we can recommend the use of LRSS6 (95% savings) in all unequal sample size cases with the possibility of improving the results if LRSS4 is used instead for large ($k \geq 20$) systems.

For the case of equal sample sizes, the results of the previous report were completely different, with the case of equal sample size, n_i=15 or 20, being different from the case with larger sample sizes, n_i=30 or 40. The results in Tables 4-6 are for equal sample sizes and are in two parts for each k, part A is for n_i=15 and 20, while part B is for n_i=30 and 40; the system reliabilities are the same as for the Tables 1-3.

Equal Sample Sizes

	LRSS1	LRSS2	LRSS3	LRSS4	LRSS5	LRSS6
Exact	7 (.1227)	7 (.1178)	7 (.1096)	7 (.1062)	5 (.0982)	5 (.0930)
.01	8 (.1313)	7 (.1264)	7 (.1180)	7 (.1145)	7 (.1061)	5 (.1005)
.02	8 (.1401)	8 (.1352)	8 (.1266)	7 (.1230)	7 (.1144)	7 (.1086)
Not a Bound	0	0	0	1	1	1
Rank	4	3	2	1	5	6

TABLE 4A: k=6

	LRSS1	LRSS2	LRSS3	LRSS4	LRSS5	LRSS6
Exact	2 (.0777)	2 (.0750)	2 (.0707)	2 (.0691)	2 (.0658)	2 (.0642)
.01	6 (.0856)	5 (.0826)	3 (.0780)	3 (.0763)	3 (.0727)	2 (.0707)
.02	8 (.0937)	8 (.0907)	6* (.0859)	5* (.0841)	4* (.0801)	4* (.0781)
Not a Bound	0	0	2	3	4	4
Rank	2	1	3	4	6	5

TABLE 4B: k=6

◯ -Designated Type of Bound (90%)
* -Not Really a .02 Bound
()-Average Standard Deviation

Equal Sample Sizes

	LRSS1	LRSS2	LRSS3	LRSS4	LRSS5	LRSS6
Exact	5 (.1133)	4 (.1057)	4 (.0926)	4 (.0871)	3 (.0730)	3 (.0647)
.01	7 (.1206)	5 (.1137)	5 (.1003)	4 (.0945)	4 (.0782)	3 (.0701)
.02	7 (.1291)	7 (.1221)	6* (.1083)	5* (.1023)	4* (.0866)	4* (.0764)
Not a Bound	1	1	2	3	4	4
Rank	2	1	3	4	6	5

TABLE 5A: k=10

	LRSS1	LRSS2	LRSS3	LRSS4	LRSS5	LRSS6
Exact	3 (.0769)	2 (.0722)	1 (.0644)	0 (.0616)	0 (.0562)	0 (.0541)
.01	5 (.0844)	4 (.0797)	1 (.0710)	1 (.0679)	0 (.0612)	(.0580)
.02	7 (.0924)	6* (.0880)	5* (.0984)	3* (.0749)	1* (.0674)	1* (.0634)
Not a Bound	1	2	3	5	7	7
Rank	1	2	3	4	6	5

TABLE 5B: k=10

O -Designated Type of Bound (90%)
* -Not Really a .02 Bound
()-Average Standard Deviation

Equal Sample Sizes

	LRSS1	LRSS2	LRSS3	LRSS4	LRSS5	LRSS6
Exact	5 (.1129)	3 (.1030)	3 (.0832)	2 (.0751)	1 (.0558)	1 (.0454)
.01	5 (.1210)	5 (.1108)	3 (.0904)	3 (.0823)	2 (.0607)	1 (.0478)
.02	5 (.1294)	5 (.1190)	5 (.0981)	3* (.0892)	3* (.0666)	2* (.0516)
Not a Bound	1	1	1	3	3	4
Rank	2	1	3	5	4	6

TABLE 6A: k=20

	LRSS1	LRSS2	LRSS3	LRSS4	LRSS5	LRSS6
Exact	2 (.0734)	1 (.0675)	0 (.0554)	0 (.0504)	0 (.0414)	0 (.0414)
.01	3 (.0796)	2 (.0745)	1 (.0611)	1 (.0552)	0 (.0428)	0 (.0397)
.02	4* (.0887)	4* (.0821)	3* (.0678)	1* (.0613)	1* (.0463)	0* (.0403)
Not a Bound	2	2	3	5	5	6
Rank	2	1	3	5	4	6

TABLE 6B: k=20

O -Designated Type of Bound (90%)
* -Not Really a .02 Bound
()-Average Standard Deviation

Again, the results for these cases are different than that for the unequal sample size case since we can see in Tables 4-6 that the higher the level of savings, the smaller the average sample-standard deviation when everything else is constant. In most cases there is a large difference: for k=10, n=15 and 20, δ=.02, the average sample standard deviations are .1291, .1221, .1083, .1023 .0866, and .0764. Unfortunately, increasing the level of savings has an adverse effect on the ability of the method to achieve a δ-exact bound: the higher the level of savings the more times the method fails to be each of the types of bounds. In many cases only LRSS1 and LRSS2 achieve any type of bound over the set of simulations. For large sample size cases, this pattern was especially pronounced as LRSS1 was a .02-exact bound 19 out of 22 times.

Thus, for systems which will be tested with equal sample sizes for all components we recommend that LRSS1 or LRSS2 be used in preferrence to the other methods.

IV. PSEUDO-SERIES RESULTS

For the pseudo-series cases, we have compared five methods of estimating lower confidence bounds, four likelihood ratio methods and the modified log-gamma method (MLGB). Three of the likelihood ratio methods, BIN9, STN13 and LRLOCK, as well as MLGB, were used for the series case (see [7]) and were discussed earlier in this report as well as in [7]. Recall

that the method LRLOCK adjusts the number of failures of any component, i, which has failed zero tests by letting the number of failures be the fraction $1/4kn_i$, where k is the number of components in the system, and n_i is the number of times the component was tested. The fourth likelihood ratio method we considered was LRL2, which replaces k in the above formula with $\sum_{i=1}^{k} \alpha_i$ where α_i is the exponent of the i-th component in the pseudo-series system. (For the series case, all $\alpha_i=1$, so this method would be identical to LRLOCK.)

The results from simulations for the pseudo-series reliability structures are presented in the followin tables. Here we have considered two cases: the case with all components in a system being from one particular set of manufacturors (one subpopulation case); the case with some of the components in the structure having been redesigned or having been manufactured by two distinct processes or manufacturers (two subpopulation case).

For both the one subpopulation and the two subpopulation cases, we have considered the situation where system reliability is of the form

$$R=P_1P_2P_3P_4P_5P_6P_7^{\alpha},$$

where α has a value of 2.5, 4.5, or 7. The corresponding test sample sizes n_i, i=1,2,3,...,7 are

$$n_i=\begin{cases} 10 & \text{for } i=1,2,3,\ldots,6 \\ (2\alpha+1)\times 10 & \text{for } i=7 \end{cases}$$

When there are two subpopulations, components which have not

been redesigned have sample sizes of $2n_i$, where n_i is as defined above. Those components which have been redesigned each have sample sizes of n_i. For both the one and two subpopulation cases, the results have been grouped into system reliability levels of .75-.80, .80-.85, .85-.90, and .90-.95. Since we observed no significant differences for the different α's, we did not separate the results as to different values of α. In the two subpopulation case, the proportion of the total population with each version of the redesigned component (or components) was an input parameter. Since we saw no noticeable differences due to particular proportions, we did not separate our results by the values of the proportions in each set of simulations. The tables indicate the number of simulation sets in which each of the five methods produced exact, δ=.01, δ=.02 bounds, and how many times each method was not any of these. The small numbers near the circled ones indicate the average values of the standard deviations over the simulation sets.

Case I: 1 Subpopulation

a) Reliabilities .75-.80, 12 simulation sets

	LRLOCK	LRL2	STN13	BIN9	MLGB
Exact	12 (.221)	9	6	7	2
.01	12	10	8	7	4
.02	12	11 (.233)	9* (.209)	8* (.213)	9* (.218)
Not a Bound		1	3	4	3
Rank	1	2	3	5	4

Case I: 1 Subpopulation

b) Reliabilities .80-.85, 13 simulation sets

	LRLOCK	LRL2	STN13	BIN9	MLGB
Exact	12 (.216)	8	1	1	1
.01	13	10	1	1	1
.02	13	13 (.227)	1 (.202)	2 (.206)	1 (.208)
Not a Bound			12	11	12
Rank	1	2	4	3	5

c) Reliabilities .85-.90, 13 simulation sets

	LRLOCK	LRL2	STN13	BIN9	MLGB
Exact	13 (.211)	13 (.203)	9	10	1
.01	13	13	12 (.184)	12 (.187)	7
.02	13	13	13	13	9* (.209)
Not a Bound				4	
Rank	5	3	1	2	4

○-Designated Type of Bound (90%)
*-Not Really a .02 Bound
()-Average Standard Deviation

Case I: 1 Subpopulation

d) Reliabilities .90-.95, 12 simulation sets

	LRLOCK	LRL2	STN13	BIN9	MLGB
Exact	12 (.194)	12 (.184)	12 (.158)	12 (.162)	12 (.150)
.01	12	12	12	12	12
.02	12	12	12	12	12
Not a Bound					
Rank	5	4	2	3	1

e) Combined Reliabilities .75-.85, 25 simulation sets

	LRLOCK	LRL2	STN13	BIN9	MLGB
Exact	24 (.219)	17	7	8	3
.01	25	20	9	8	5
.02	25	24 (.230)	10* (.206)	10* (.210)	10* (.213)
Not a Bound		1	15	15	15
Rank	1	2	3	4	5

○ -Designated Type of Bound (90%)
* -Not Really a .02 Bound
()-Average Standard Deviation

If a person wants to predict a 90% lower confidence bound, and he has a system similar to the one from which these results were gathered, he should use the results presented to determine which of these methods seems most useful. Which table to use is determined by the insight the user has on the reliability of the system. If he feels for instance, that the system's true reliability is between .85 and .90, he could use Table b) to see that the method STN13 should be used on his data as a δ=.01 bound. If however, the engineer could only say that he felt the true reliability was between .75 and .95, he would have to use Table g), which indicates that LRLOCK should be used as an exact bound.

Some information is lost in the second example above. Notice in Tables e) and f), that LRLOCK appears to be the best method to use for the reliabilities from .75 to .85, but for reliabilities between .85 and .95, LRL2, STN13 and BIN9 appear to be better methods. Thus we see what information may be lost when a rougher guess of the system's reliability must be made.

The following tables will give the results for the two subpopulation case.

Case I: 1 Subpopulation

f) Combined Reliabilities .85-.95, 25 simulation sets

	LRLOCK	LRL2	STN13	BIN9	MLGB
Exact	25 (.203)	25 (.203)	21	22	13
.01	25	25	24 (.194)	24 (.186)	19
.02	25	25	25	25	21* (.188)
Not a Bound					4
Rank	4	3	2	1	5

g) Combined Reliabilities .75-.95, 50 simulation sets

	LRLOCK	LRL2	STN13	BIN9	MLGB
Exact	49 (.211)	42	28	30	16
.01	50	45 (.212)	33	32	24
.02	50	49	35* (.193)	35* (.197)	31* (.200)
Not a Bound			15	15	19
Rank	1	2	3	4	5

O -Designated Type of Bound (90%)
* -Not Really a .02 Bound
()-Average Standard Deviation

Case II: 2 Subpopulations

a) Reliabilities .75-.80, 12 simulation sets

	LRLOCK	LRL2	STN13	BIN9	MLGB
Exact	10	9	4	5	7
.01	12 (.166)	11 (.164)	7	10	10
.02	12	12	11 (.159)	11 (.165)	11 (.170)
Not a Bound			1	1	1
Rank	4	2	1	3	5

b) Reliabilities .80-.85, 12 simulation sets

	LRLOCK	LRL2	STN13	BIN9	MLGB
Exact	8	7	3	5	6
.01	12 (.166)	10	6	7	9
.02	12	12 (.164)	10* (.159)	10* (.165)	10* (.170)
Not a Bound			2	2	2
Rank	2	1	3	4	5

○ -Designated Type of Bound (90%)
* -Not Really a .02 Bound
()-Average Standard Deviation

Case II: 2 Subpopulations

c) Reliabilities .85-.90, 17 simulation sets

	LRLOCK	LRL2	STN13	BIN9	MLGB
Exact	10	9	8	8	1
.01	11	12	8	10	6
.02	17 (.154)	17 (.152)	17 (.140)	17 (.142)	17 (.146)
Not a Bound					
Rank	5	4	1	2	3

d) Reliabilities .90-.95, 12 simulation sets

	LRLOCK	LRL2	STN13	BIN9	MLGB
Exact	11	7	6	7	0
.01	11	10	7	8	2
.02	11	10* (.138)	9* (.125)	10* (.130)	6* (.123)
Not a Bound	1	2	3	2	6
Rank	1	3	4	2	5

◯ -Designated Type of Bound (90%)
* -Not Really a .02 Bound
()-Average Standard Deviation

Case II: 2 Subpopulations

e) Combined Reliabilities .75-.85, 24 simulation sets

	LRLOCK	LRL2	STN13	BIN9	MLGB
Exact	18	16	7	10	13
.01	24 (.163)	21	13	17	19
.02	24	21* (.170)	21* (.153)	21* (.162)	21* (.167)
Not a Bound			3	3	3
Rank	1	2	3	4	5

f) Combined Reliabilities .85-.95, 29 simulation sets

	LRLOCK	LRL2	STN13	BIN9	MLGB
Exact	21	16	14	15	1
.01	22	22	15	18	8
.02	28 (.148)	27 (.145)	26 (.133)	27 (.136)	23* (.135)
Not a Bound	1	2	3	2	6
Rank	4	3	1	2	5

○ -Designated Type of Bound (90%)
* -Not Really a .02 Bound
()-Average Standard Deviation

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Case II: 2 Subpopulations

g) Reliabilities .75-.95, 53 simulation sets

	LRLOCK	LRL2	STN13	BIN9	MLGB
Exact	38	32	21	25	14
.01	46	43	28	35	27
.02	52 (.160)	51 (.158)	47* (.145)	48 (.149)	44* (.146)
Not a Bound	1	2	6	5	9
Rank	3	2	4	1	5

◠ -Designated Type of Bound (90%)
* -Not Really a .02 Bound
()-Average Standard Deviation

These tables are used just as those for the one subpopulation case.

Let us now look at what problems may occur if a user follows the tables after making an incorrect guess of the system reliability. If we choose a method which works (produces a 90% bound a high percent of the time) both at the true system reliability and at the guessed reliability, we will get a valid bound. If the method works at the true reliability level of the system, but we would have used another method had we made a correct guess, we will be worse off because another method may have predicted a bound which is closer to the true reliability. An error could be made however,

method was seen not to produce a true 90% lower bound
e correct system reliability. Thus, it may be useful
ake a close look at the reasoning behind predicting the
tem's reliability.